Sample Question Paper
Applied Mathematics (Code-241)
Class XII
2023-24

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A,B,C,D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer(VSA)** questions of 2 marks each.
4. **Section C** has 6 **Short Answer(SA)** questions of 3 marks each.
5. **Section D** has 4 **Long Answer(LA)** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**04 marks each**)** with sub parts.
7. Internal Choice is provided **in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D.** You have to attempt only one alternatives in all such questions.

SECTION A
(All Questions are compulsory. No internal choice is provided in this section)

Q -1 The value of -70 mod 13 is
(a) 5 (b) -5 (c) 8 (d)-8

Q -2 If $\frac{x+1}{x+2} \geq 1$, then
(a) $x \in [-\infty, 2]$ (b) $x \in (-\infty, -2)$ (c) $x \in (-\infty, 2]$ (d) $x \in (-\infty, 2)$

Q-3 Which of the following is a statistic
(a) μ (b) $\bar{x}$ (c) σ^2 (d) None

Q-4 In one sample test, the estimation for population mean is
(a) $\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ (b) $\frac{\bar{x}-\mu}{s/n}$ (c) $\frac{\bar{x}-\mu}{s^2/n}$ (d) None

Q-5 A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. Then the rate of the stream is
(a) 2 km/hr (b) 4 km/hr (c) 6 km/hr (d) 8 km/hr

Q-6 If random variable X represents the number of heads when a coin is tossed twice then mathematical expectation of X is
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

Q-7 The least non-negative remainder when 3^{50} is divided by 7 is
(a) 4 (b) 3 c) 30 d) 1

Q-8 If the cash equivalent of a perpetuity of Rs 300 payable at the end of each quarter is Rs 24000 then rate of interest converted quarterly is
(a) 5% (b) 4% (c) 3% (d) 2%

Q-9 The value of $\int \frac{logx}{x} dx$ is

(a) $\frac{logx}{2} + C$ (b) $\frac{(logx)^2}{2}$ +C (c) $logx$ + C (d) None

Q-10 The supply of finished good was delayed for a month due to landslide in hilly terrain. Under which trend oscillation does this situation fall

(a) $Seasonal$ (b) $Cyclical$ (c) $Secular$ (d) $Irregular$

Q-11 A machine costing ₹ 30,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 4000. The annual depreciation is

(a) ₹ 5500 (b) ₹ 6500 (c) ₹ 7500 (d) ₹ 8500

Q-12 The effective rate of interest equivalent to the nominal rate 6% compounded semi-annually is

(a) 6.05% (b) 6.07% (c) 6.09% (d) None

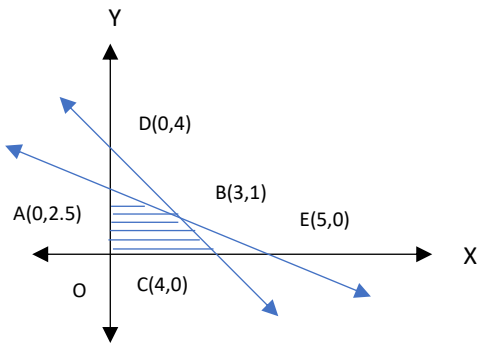
Q-13 If the investment of ₹ 20000 in the mutual fund in 2015 increased to ₹ 32000 in year 2020, then CAGR (Compound Annual Growth rate is) is [Given$(1.6)^{\frac{1}{5}} = 1.098$]

(a) 9.08% (b) 9.8% (c) 0.098 (d) 0.09

Q-14 The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^3$ $(x \neq 0)$ is

(a) x (b) $logx$ (c) x^2 (d) $\frac{1}{x^2}$

Q-15 Besides non negativity constraint the figure given below is subject to which of the following constraints



(a) $x + 2y \leq 5 ; x + y \leq 4$ (b) $x + 2y \geq 5 ; x + y \leq 4$

(c) $x + 2y \geq 5 ; x + y \geq 4$ (d) $x + 2y \leq 5 ; x + y \geq 4$

Q-16 If X is a Poisson variate such that 3P(X=2) = 2P(X=1) then the mean of the distribution is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $-\frac{4}{3}$ (d) $-\frac{3}{4}$

Q-17 For the given five values 35, 70, 36, 59, 64, the three years moving averages are given by

$(a)47, 53, 55$ (b) $53, 47, 45$ (c) $47, 55, 53$ (d) 45, 55, 57

Q-18 The data point of a normal variate with mean 12, standard deviation 4 and Z – score 5 is

(a) 28 (b) 304 (c) 34 (d) 32

ASSERTION REASON BASED QUESTIONS

In the following questions, a statement of Assertion(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a. Both A and R are true and R is the correct explanation of A.
b. Both A and R are true and R is not the correct explanation of A.
c. A is true but R is false.
d. A is false and R is true.

Q-19 Assertion (A): The maximum profit that a company makes if profit function is given by $P(x) = 41 + 24x - 8x^2$; where 'x' is the number of units and P is the profit in rupees is 59

Reason (R) : The profit is maximum at x = a if $P'(a) = 0 \; and \; P''(a) > 0$

Q-20 Assertion (A) : The probability of getting 6 heads when a unbiased coin is tossed 10 times is $C(10,6)\left(\frac{1}{2}\right)^{10}$

Reason (R) In a Binomial distribution the probability is given by $P(X= r) = C(n, r)(p)^r(q)^{n-r}$

SECTION B

All Questions are compulsory. In case of internal Choice, attempt any one question only

Q-21 At what rate of interest will the present value of perpetuity of Rs 1500 payable at the end of every 6 months be Rs 20,000?

Q-22 If A is a square matrix $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ such that $A^2 = pA$, then find the value of p.

OR

If $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew -symmetric, matrix, then find value of $a+b+c$

Q-23 A Cooperative Society of farmers has 10 hectares of land to grow two crops A and B. To control weeds, pesticide has to be used for crops A and B at the rate of 30 grams and 15 grams per hectare. Further, no more than 750 grams of pesticide should be used. The profit from crops A and B per hectare are estimated as Rs 8000 and Rs 9500. Formulate the above problem as LPP, in order to allocate land to each crop for maximum use.

Q-24 A boatman takes twice as long as to go upstream to a point as to return downstream to the starting point. If the speed of a boat in still water is 15 km/hr, what is the speed of the stream.

OR

'A' can run 40 meters while 'B' runs 50 meters in the same time. In a 1000 m race, find by how much distance 'B' beats 'A'.

Q-25 A machine produces washers of thickness 0.50mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53mm and the standard deviation is 0.03mm. Test the hypothesis at 5% level of significance that the machine is working in proper order.
[Given $t_{0.025} = 2.262 \; at \; 9 \; degree \; of \; freedom$]

SECTION C

All Questions are compulsory. In case of internal Choice, attempt any one question only

Q-26 Evaluate: $\int \frac{x^3}{(x+2)} dx$

OR

Evaluate: $\int (x^2 + 1) \log_e x \; dx$

Q-27 Cost of two toys A and B are Rs 50 and Rs 75. On a particular Sunday shopkeeper P sells 7 toys of type A and 10 toys of type B whereas shopkeeper Q sells 8 toys of type A and 6 toys of type B. Find income of both shopkeepers using matrix Algebra.

Q-28 Find the intervals in which the function f(x) = $2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.

Q-29 The demand and supply functions under the pure market competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find **Consumer's surplus**.

OR

The demand and supply functions under the pure market competition are $p_d = 56 - x^2$ and $p_s = 8 + \frac{x^2}{3}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find **Producer's surplus.**

Q-30 Mr Surya borrows a sum of Rs. 5,00,000 with total interest paid Rs 2,00,000(flat) and he is paying an EMI of Rs. 12,500. Calculate loan tenure.

Q-31 Mr Sharma wants to send his daughter abroad for higher studies after 10 years. He sets up a sinking fund in order to have Rs. 500,000 after 10 years. How much should he set aside bi-annually into an account paying 5% per annum compounded annually.
[Use $(1.025)^{20} = 1.6386$]

SECTION D

(This section comprises of long answer type questions (LA) of 5 mark each)

Q-32 On doing the proof reading of a book on an average 4 errors in 10 pages were detected. Using Poisson's distribution find the probability of (i) No error and (ii) one error in 1000 pages of first printed edition of the book (Given $e^{-0.4} = 0.6703$)

OR

How many time must Sunil toss a fair coin so that the probability of getting at least one head is more than 90 %

Q-33 A manufacturer has three machines I,II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. He produces only two items M and N , each requiring the use of all the three machines. The number of hours required for producing 1 unit of M and N on three machines are given in the following table:

Items	**Number of hours required on machines**		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of Rs 600 and Rs 400 on one unit of items M and N respectively. How many units of each item be produced so as to maximize the profit . What is the maximum Profit?

Q-34 A company produces a certain commodity with Rs 2400 fixed cost. The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of Rs 8 per unit. Find the following

(i) Cost Function. (ii) Revenue Function

(iii) Breakeven Point (iv) Profit Function

OR

The production manager of a company plans to include 180 sq. cms. of actual printed matter in each page of a book under production. Each page should have a 2.5 cm wide margin along the top and bottom and 2 cm wide margin along the sides. What are the most economical dimensions of each printed page?

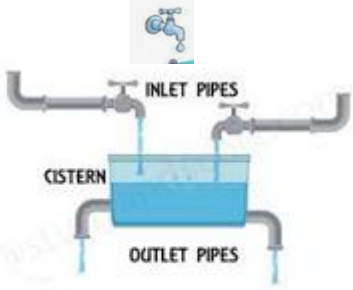
Q-35 The management committee of a Welfare Club decided to award some of its members (say x) for sincerity, some (say y) for helping others selflessly and some others (say z) for effective management. The sum of all the awardees is 12. Three times the sum of all awardees for helping others selflessly and effective management added to two times the number of awardees for sincerity is 33. If the sum of the number of awardees for sincerity and effective management is twice the number of awardees for helping others, use matrix method to find the number of awardees of each category.

SECTION E

(This section comprises of 3 source based questions (Case Studies) of 4 mark each)

Q-36 Case Study 1 : Pipes and Cisterns (Mark 2+1+1) (Internal choice is in the iii part)

A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 Hrs. B and C together fill the tank in 10 hours. A and C together fill the tank in $7\frac{1}{2}\ hrs$. Based on above information answer the following questions.



(i) In how much time will A, B and C fill the tank?
(ii) In how much time will A separately fill the tank?
(iii) In how much time will B separately fill the tank?

OR

In how much time will C separately fill the tank?

Q-37 Case Study 2: Read the following passage and answer the questions below (**Internal Choice is in option iii**.) (Mark 1 + 1 + 2)

Let X denote the number of hours a person watches television during a randomly selected day. The probability that X can take the values x_i, has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & otherwise \end{cases}$$

(i) Find the value of k.

(ii) What is the probability that a person watches two hours of television on a selected day?

(iii) What is the probability that the person watches at least two hours of television on a selected day ?

OR

(iv) What is the probability that the person watches at most two hours of television on a selected day ?

Q-38 Case Study 3 :

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production

The table below shows the welfare expenses(in lakh Rs) of Steel Industry during 2001-2005. Fit a straight line trend by the method of least squares and estimate the trend for the year 2008.

Year	2001	2002	2003	2004	2005
Welfare expenses	160	185	220	300	510

OR

The annual rainfall(in mm) was recorded in Cherrapunji, Meghalaya

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Rainfall(in mm)	1.2	1.9	2	1.4	2.1	1.3	1.8	1.1	1.3

Determine the trend of rainfall by three years moving average and draw the moving averages graph.

Applied Mathematics (241)
Marking Scheme
Class XII (2023-24)

Section A
(1 Mark each)

Q-1 Option (c) Here X < 0 and Y > 0 , hence -70 mod 13 is 8 $\because 8 > 0$. 1 Mark

Q-2 Option (b) $x \in (-\infty, -2)$ 1 Mark

$\frac{x+1}{x+2} \geq 1 \Longrightarrow \frac{x+1}{x+2} - 1 \geq 0$

$\Longrightarrow \frac{x+1-x-2}{x+2} \geq 0$

$\Longrightarrow \frac{-1}{x+2} \geq 0 \Longrightarrow$ x+2 < 0 $\left[\because \frac{a}{b} > 0 \; and \; a < 0 \Longrightarrow b < 0\right]$

$\Longrightarrow x < -2$

Q-3 Option (b) $\bar{x}$ *is a* statistic 1 Mark
Q-4 Option (a) 1 Mark
Q-5 Option (a)

Let man's rate upstream = x km/hr
Let man's rate downstream = 2x km/hr
Hence , Man's rate in still water = $\frac{1}{2}(x + 2x)$ = $\frac{3x}{2}$ km/hr
Therefore $\frac{3x}{2} = 6 \Longrightarrow x = 4 \; km/hr$
Man's rate downstream = 8 km/hr
Hence rate of stream $\frac{1}{2}(8 - 4) = 2 \; km/h$ 1 Mark

Q-6 Option (d)

x_i	Sample Event	P(x_i) = p_i	$x_i p_i$
0	TT	$\frac{1}{4}$	0
1	HT,TH	$\frac{1}{2}$	$\frac{1}{2}$
2	HH	$\frac{1}{4}$	$\frac{1}{2}$

Mathematical Expectation E(X) = $\sum p_i x_i$ = 1 1 Mark

Q-7 Option (c) $3^1 \equiv 3(mod7) \Longrightarrow 3^2 \equiv 3 \times 3 = 2(mod7)$
$\Longrightarrow 3^3 = 3 \times 2 = 6 = -1(mod7)$
$\Longrightarrow (3^3)^{16} = (-1)^{16}(mod7)$
$\Longrightarrow (3^3)^{16} = 1(mod7) \Longrightarrow (3^3)^{16} \times 3^2 = 1 \times 3^2(mod7)$
$\Longrightarrow 3^{50} = 2(mod7)$ 1 Mark

Q-8 Option (a) i = $\frac{r}{400}$.

P = $\frac{R}{i} \Longrightarrow 24000 = \frac{300\times400}{r} \Longrightarrow r = \frac{120}{24} = 5\%$ 1 Mark

Q-9 Option (b) $\int \frac{logx}{x} dx$

Put log x = t

Differentiating $\frac{1}{x}dx = dt$

Hence, $\int \frac{logx}{x} dx = \int tdt = \frac{t^2}{2} + C = \frac{(logx)^2}{2} + C$ 1 Mark

Q-10 Option (d) 1 Mark

Q-11 Option (b) 1 mark

D = $\frac{C-S}{n} \Longrightarrow D = \frac{30{,}000-4000}{4} = \frac{26000}{4} = 6500$

Hence, the depreciation is Rs. 6500 1 Mark

Q-12. Option (c)

$$r_{eff} = \left[\left(1+\frac{r}{m}\right)^m - 1\right] \times 100$$

$$r_{eff} = [(1.03)^2 - 1] \times 100 = (1.0609 - 1) \times 100 = 6.09\%$$

Q-13 Option (b)

$$\text{CAGR} = \left[\left(\frac{EV}{SV}\right)^{\frac{1}{5}} - 1\right] \times 100$$

$$= \left[\left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1\right] \times 100$$

$$= \left[(1.6)^{\frac{1}{5}} - 1\right] \times 100$$

$$= [1.098 - 1] \times 100 = 0.098 \times 100 = 9.8\%.$$ 1 Mark

Q-14 Option (c) $x\frac{dy}{dx} + 2y = x^3$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{x^3}{x}$

$\frac{dy}{dx} + \frac{2y}{x} = x^2$

I.F = $e^{\int \frac{2}{x}} = e^{2lnx} = e^{lnx^2} = x^2$ 1 Mark

Q-15 Option (a) 1 Mark

Q-16 Option (a) 1 Mark

3P(X = 2) = 2P(X = 1)

$\Longrightarrow 3\frac{m^2e^{-m}}{2!} = 2\frac{me^{-m}}{1!} \Longrightarrow m = \frac{4}{3}$

Q-17 Option (c) 1 Mark

Q-18 Option (d)

Z = $\frac{x-\mu}{\sigma} \Longrightarrow 5 = \frac{x-12}{4} \Longrightarrow x = 32.$ 1 Mark

Q-19 Option(c)

Assertion : P(x) = 41 + 24x-8x^2

$P'(x) = 24 - 16x$

$P'(x) = 0 \Longrightarrow 24 - 16x = 0 \Longrightarrow x = \frac{24}{16} = \frac{3}{2}$

$P''(x) = -16 < 0 \Longrightarrow x = \frac{3}{2}\ is\ a\ point\ of\ maxima$

Max Profit = P = 41+ $24 \times \frac{3}{2} - 8 \times \frac{9}{4} = 41 + 36 - 18 = 59$

Assertion is true but Reason is false , for Maximum $P'(x) = 0\ and\ P''(x) < 0.$ 1 Mark

Q-20 Option (a) Both A and R are true and R is the correct explanation of A 1 Mark

Section B
(2 Marks each)

Q-21 Let rate of interest be r% per annum, then i = $\frac{r}{200}$

Given R = Rs 1500 and P = Rs 20,000

P = $\frac{R}{i} \Longrightarrow i = \frac{R}{P} = \frac{1500}{20000}$ 1 Mark

$\Longrightarrow \frac{r}{200} = \frac{1500}{20000} \Longrightarrow r = 15\%$ 1 Mark

Q-22 A = $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$ 1 Mark

$A^2 = pA \Longrightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$\Longrightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \Longrightarrow p = 4$ 1 Mark

OR

$$\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix}$$ ½ Mark

Comparing a = -2 ; b = -b $\Longrightarrow 2b = 0 \Longrightarrow b = 0 \; and \; c = -3$ 1½ Mark

Hence a+b+c = -2+0-3 = -5. ½ Mark

Q-23 Let 'x' hectares and 'y' hectares of land be allocated to crop A and Crop B

Max Z = 8000x + 9500y. ½ Mark

Subject to $x + y \leq 10$; $2x + y \leq 50; x \geq 0 \; and \; y \geq 0$ 1½ mark

Q-24 $\frac{time\ taken\ upstream}{time\ taken\ downstream} = \frac{2}{1}$ 1 Mark

Let speed of boat = 15 km/hr and speed of stream = y km/hr.

Hence $\frac{15+y}{15-y} = \frac{2}{1}$ ½ Mark

$\Longrightarrow 15 + y = 30 - 2y.$

$\Longrightarrow$ 3y = 15 $\Longrightarrow y = 5 \; Km/hr$ ½ Mark

OR

When B runs 50 m A runs 40 m ½ Mark

When B runs 1 m , A runs = $\frac{40}{50} = \frac{4}{5}$ ½ Mark

When B runs 1000 m , A runs = $\frac{4}{5} \times 1000$ = 800 m ½ Mark

Hence B beats A by 200 m ½ Mark

Q-25 Define Null hypothesis H_0 alternate hypothesis H_1 as follows:

H_0: μ = 0.50 mm

H_1 : $\mu \neq 0.50 \; mm$

Thus a two-tailed test is applied under hypothesis H_0, we have

$t = \frac{\bar{X}-\mu}{s}\sqrt{n-1} = \frac{0.53-0.50}{0.03} \times 3 = 3.$ 1 Mark

Since the calculated value of t i.e. t_{cal}(=3) > t_{tab}(=2.262), the null hypothesis H_0 can be rejected. Hence, we conclude that machine is not working properly. 1 Mark

Section C
(3 Marks each)

Q-26 $\int \frac{x^3}{(x+2)} dx = \left(\int x^2 - 2x + 4 - \frac{8}{x+2}\right) dx$ 2 Mark

$= \frac{x^3}{3} - x^2 + 4x - 8\ln(x+2) + C.$ 1 $Mark$

(where C is an arbitrary constant of integration)

OR

$\int (x^2+1) ln\ x dx$

Integrating by parts

$lnx\left(\frac{x^3}{3} + x\right) - \int \frac{1}{x}\left(\frac{x^3}{3} + x\right) dx.$ 2 mark

$lnx\left(\frac{x^3}{3} + x\right) - \int \left(\frac{x^2}{3} + 1\right) dx$

$lnx\left(\frac{x^3}{3} + x\right) - \left(\frac{x^3}{9} + x\right)$ + C. 1 Mark

Q-27

Toy A. Toy B

$\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix}$ Here Row 1 and Row 2 indicate Shopkeeper 1 and Shopkeeper 2

Cost Matrix = $\begin{bmatrix} 50 \\ 75 \end{bmatrix}$ 1 Mark

Amount = $\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix}\begin{bmatrix} 50 \\ 75 \end{bmatrix} = \begin{bmatrix} 350 + 750 \\ 400 + 450 \end{bmatrix} = \begin{bmatrix} 1100 \\ 850 \end{bmatrix}$

Income of Shopkeeper P is Rs 1100/ and shopkeeper Q is Rs 850/ 2 Marks

Q-28 f(x) = $2x^3 - 9x^2 + 12x - 5$

$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$f'(x) = 6(x-1)(x-2)$ 1 Mark

$f'(x) = 0 \Longrightarrow x = 1\ and\ x = 2\ are\ the\ critical\ points.$ ½ Mark

The intervals are $(-\infty, 1)$; $(1,2)$; $(2, \infty)$ ½ Mark

Increasing in $(-\infty, 1) \cup (2, \infty)$ Decreasing in $(1,2)$ 1 Mark

Q-29 Under pure competition

$p_d = p_s$

$\Longrightarrow 16 - x^2 = 2x^2 + 4$

$\Longrightarrow 3x^2 = 12$ $\Longrightarrow$ x = 2 , -2 ; since x can't be –ve, so x=2 1 Mark

When $x_0 = 2$; $p_0 = 12$ ½ Mark

Hence, Consumer's surplus = $\int_0^2 p_d dx - p_0 x_0$ ½ Mark

$= \int_0^2 (16 - x^2)\, dx - 12 \times 2$

= 16/3 units 1 Mark

OR

$p_d = p_s$

$\Longrightarrow 56 - x^2 = 8 + \frac{x^2}{3}$

$\Longrightarrow \frac{4}{3}x^2 = 48 \Longrightarrow x^2 = 36 \Longrightarrow$ x = 6 , -6 ; since x can't be –ve, so x=6 1 Mark

When $x_0 = 6\ ;\ p_0 = 20$ ½ Mark

Hence, Producer's surplus = $p_0 x_0 - \int_0^6 p_s dx$ ½ Mark

$= 6 \times 20 - \int_0^6 \left(8 + \frac{X^2}{3}\right) dx$

= 120 – [48+24]

= 48 units 1 Mark

Q-30 Here P = 5,00,000 ; I = 2,00,000; EMI = 12,500

EMI = $\frac{P+I}{n}$ 1½ Mark

12,500 = $\frac{5,00,000+2,00,000}{n} \Longrightarrow$ n = $\frac{7,00,000}{12,500}$ = 56 months. 1½ Mark

Q-31 Let Rs. R be set aside biannually for 10 years in order to have Rs. 500,000 after 10 years

Here S = 500,000. ; n = $10 \times 2 = 20$

i = $\frac{5}{2\times100} = 0.025$ ½ Mark

R = $\frac{iS}{(1+i)^n-1} = \frac{0.025\times500,000}{(1.025)^{20}-1} = \frac{12,500}{1.6386-1} = 19,574.07.$ 2½ Mark

Section D
(5 Marks each)

Q-32 Here m = 0.4

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.4}\times(0.4)^r}{r!}$

$= \frac{0.6703\times(0.4)^r}{r!}$ 1 Mark

In 1000 pages error = $1000 \times \frac{0.6703\times(0.4)^r}{r!}$ ½ Mark

For zero error $P(X = 0) = 1000 \times \frac{e^{-m}.m^0}{0!} = \frac{e^{-0.4}\times(0.4)^0}{0!}$

$= 1000 \times 0.6703 = 670.3$ 1½ Mark

For one error $P(X = 1) = 1000 \times \frac{e^{-m}.m^1}{1!} = \frac{e^{-0.4}\times(0.4)^1}{1!}$

$= 670.3 \times 0.4 = 268.12$ 2 Mark

OR

Here p = $\frac{1}{2}$ $and\ q = \frac{1}{2}$

P(X=r) = C(n, r)$p^r q^{n-r}$

1-P(r = 0) > $\frac{90}{100}$ 1 Mark

1- $C(n,0)\left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{9}{10}$

$\Rightarrow \quad \frac{n!}{0!(n-0)!}\left(\frac{1}{2}\right)^n < \frac{1}{10}$ 2 Mark

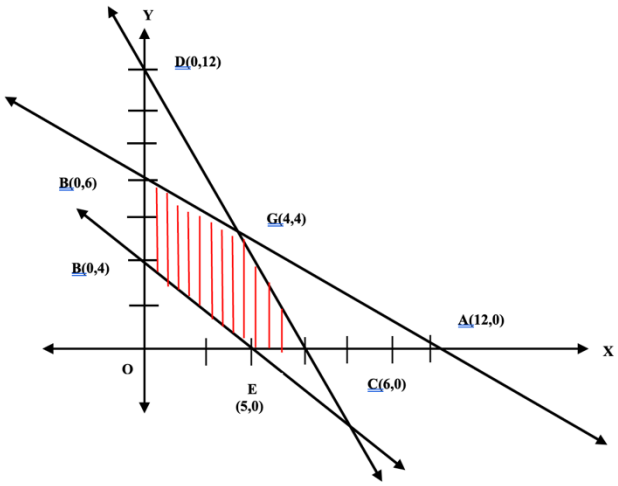
$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{10} \Rightarrow 2^n > 10 \Rightarrow n\ is\ 4\ or$ more times 2 Mark

Q-33 Let 'x' and 'y' be the number of units of items M and N respectively.

We have : $x \geq 0, y \geq 0$

$x + 2y \leq 12 ; 2x + y \leq 12 ; \ x + \frac{5}{4}y \geq 5.$ 1½ Mark

Max Z = 600x + 400 y 1 Mark



Graph 1½ Mark

Corner Point	**Z = 600x + 400y**
E : (5,0)	3000
C : (6,0)	3600
G : (4,4)	**4000 (Maximum)**
B : (0,6)	2400
F : (0,4)	1600

Hence maximum profit is Rs 4000 when 4 units of each of the items M and N are produced. 1 Mark

Q-34. Let 'x' units of product be produced and sold. As selling price of one unit is Rs 8 total revenue on 'x' units = Rs 8x

(i) Cost Function C(x) = Fixed Cost + 25% of 8x

$= 24000 + \frac{25}{100} \times 8x$

= 24000 +2x. 1 ½ Mark

(ii) Revenue Function = 8x 1 Mark

(iii) Breakeven Point 8x = 24000+2x

x = 4000 1½ Mark

(iv) Profit function = R(x) – C(x) = 6x – 24000 1 Mark

OR

Let x and y be the dimension of the printed pages then x.y = 180.

A = Area of the page = (x+4)(y+5)

= xy +5x +4y + 20

$= 180 + 5x + 4\times\left(\frac{180}{x}\right) + 20$

$= 200 + 5x + \frac{720}{x}$ 2½ Mark

For most economical dimension $\frac{dA}{dx} = 0 \Longrightarrow 5 - \frac{720}{x^2} = 0.$

$\Longrightarrow x^2 = 144 \Longrightarrow x = 12$

Now $\frac{d^2A}{dx^2} = \frac{1440}{x^3}$

$\left(\frac{d^2A}{dx^2}\right)_{x=12} = \frac{1440}{12^3} > 0. \therefore A \text{ is minimum}$

Hence, the most economical dimensions are 16cm and 20 cm 2½ Mark

Q-35 x + y + z = 12

2x + 3y + 3z = 33

x – 2y + z = 0

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$ 1 Mark

|A| = 3 $\neq 0$ ½ Mark

$$\text{adjA} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$ 2½ Mark

$$X = A^{-1}B \Longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$X = A^{-1}B \Longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 108 - 99 + 0 \\ 12 + 0 - 0 \\ -84 + 99 + 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Hence x = 3 , y = 4, z = 5 1 Mark

Section E
(Case Studies Based Questions)

Q- 36 **Case Study – I**

(i)

A + B fill the tank in 6 hrs

B + C fill the tank in 10 hrs

A + C fill the tank in $\frac{15}{2}$ hrs

$2(A + B + C) = \frac{6 \times 10 \times \frac{15}{2}}{6 \times 10 + 6 \times \frac{15}{2} + 10 \times \frac{15}{2}} = \frac{450}{60+45+75} = \frac{450}{180} = \frac{5}{2}\ hrs$

Hence A,B and C together will fill the tank in 5 Hrs 2 Mark

(ii) A will in $[(A+B+C) - (B+C)] = \frac{10 \times 5}{10-5} = 10\ hrs$ 1 Mark

(iii) B will fill in $\frac{\frac{15}{2} \times 5}{\frac{15}{2} - 5} = 15\ hrs$ 1 Mark

OR

C will fill in $\frac{5\times6}{6-5} = 30\ hrs$

Q-37 **Case Study – II**

x_i	0	1	2	3	4	5
P(X=x_i)	0.2	k	2k	2k	0	0

(i) Since $\sum P = 1 \implies 0.2 + k + 2k + 2k = 1 \implies 0.2 + 5k = 1 \implies 5k = --0.2$

$\implies$ k = $\frac{4}{25}$ 1½ Mark

(ii) P(X=2) = 2k = $\frac{8}{25}$ 1 Mark

(iii) $P(X \geq 2) = 4k = \frac{16}{25}$ 1½ Mark

OR

$P(X \leq 2)$ = 0.2+3k = $\frac{17}{25}$

Q-38 **Case Study – III**

Year	Y	X=Year - 2003	X^2	XY
2001	160	-2	4	-320
2002	185	-1	1	-185
2003	220	0	0	0
2004	300	1	1	300
2005	510	2	4	1020
	1375		10	815

2 Marks for table

a = $\frac{\sum Y}{n} = \frac{1375}{5} = 275$ ½ Mark

b = $\frac{\sum XY}{\sum X^2} = \frac{815}{10} = 81.5$ ½ Mark

$Y_c = a + bX$

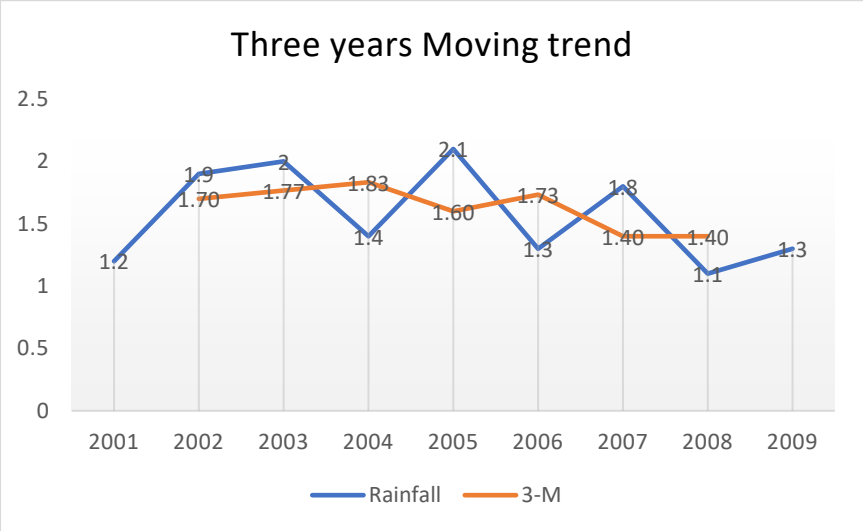
$Y_c = 275 + 81.5\ X$

The estimated value for 2008 will be $275 + 151.5 \times 5 = 275 + 757.5 = 1032.5$. 1 Mark

OR

Year	Rainfall(in cm)	3 years moving total	3 years moving average
2001	1.2		
2002	1.9	5.1	1.70
2003	2	5.3	1.77
2004	1.4	5.5	1.83
2005	2.1	4.8	1.60
2006	1.3	5.2	1.73
2007	1.8	4.2	1.40
2008	1.1	4.2	1.40
2009	1.3		

1½ Marks for table



2½ Mark Marks for graph